
Summing up the Education of Mathematically Gifted Students

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Recent government reports identify the importance of producing individuals with high-level mathematical capabilities who can contribute creatively to Australia’s intellectual capital. To address the shortfall in such individuals, mathematically gifted students must be identified and nurtured. Hence, teachers’ programs need to be well-grounded in research on educating the mathematically gifted. In this paper, we report on the status of research on these students, draw upon our research to inform practice, and highlight myths associated with the education of these students.

At the commencement of the 20th century, Australia was proclaimed to be “the paradise of mediocrity and the grave of genius” (NSW Attorney General, 1901, as cited in Barcan, 1983). A century later we hear of the need for a “clever country”, and how we now should value the role of our creative individuals: “we are seeking today to nurture a new generation of young scientific minds capable of achieving great things for their country” (Howard, 2001). However, if we are to enhance our performance in the sciences, and rhetoric become reality, we must address the education of our most able students. Thus given this focus on creative achievement, the need to redress the shortfall in students who choose to pursue mathematical studies, and the importance of mathematics to the new economy (e.g., Miles, 2000; Thomas, 2000), the education of mathematically gifted students must be a high priority.

Neglect concerning the status of the mathematical sciences has been recognised in the context of a national focus on numeracy. Through our concerted attention to the “minimums” of mathematical capability, we have been overlooking the “maximums”. MacGillivray (2000), suggests three distinctive levels of mathematical capability:

1. the quantitative capabilities of the whole of society;
2. the mathematical capabilities in the broad spectrum of areas with quantitative links;
3. the high level expertise capability of the discipline of mathematical sciences.

While numeracy is an important starting point, mathematics education must also prepare those potentially most able students to achieve the second and third levels of capabilities. The second concern relates to equitable educational provision for “ALL students ... including the average, above average and most able students” (MacGillivray, 2000). The Senate report into the education of gifted children (Collins,
2001) acknowledges that focusing on minimum standards could have a deleterious effect on satisfying the special needs of the gifted, who are already affected by “underachievement, boredom, frustration, and psychological distress, (and) ... negative attitudes and mistaken beliefs” (p. xiii). The report argues that equitable provision for these students includes teacher training in the identification of gifted students and gifted pedagogy, the development of appropriate curriculum materials, and curriculum differentiation. It also highlights the inadequacy of current provisions for the unique needs of these children: “Ad hoc enrichment activities, or enrichment for the whole class, are insufficient” (Collins, 2001). Others have recommended fostering high level capabilities by making mathematics “more exciting” and by applying “creative, innovative approaches to problem solving” (Batterham, 2000, p. 51), and by “using extension activities to supplement the normal curriculum” (Thomas, 2001 p. 21). However, such strategies in isolation are inconsistent with the research on gifted education (Collins, 2001), and inadequate because they fail to address the myriad of cognitive (e.g., knowledge), environmental (e.g., education), and personality variables (e.g, motivation) that contribute towards creative achievement (Eysenck, 1995) and which can be fostered from childhood (Torrance, 1994). Thus, to optimise a mathematically gifted individual’s chances for creative achievement, his or her needs should be addressed from the early years at school. The purpose of this paper is to examine the status of research on mathematically gifted students and to explore some of the assumptions underpinning the education of mathematically gifted children. First, we provide an overview on the education of mathematically gifted students. Second, we examine Australasian literature on mathematically gifted students, and third, we suggest key directions based on our research.

The Education of Mathematically Gifted Students

Mathematically gifted students are distinguished from their non-gifted peers by their exceptional reasoning ability (House, 1987; M. Johnson, 1983). These students may also exhibit an exceptional memory, the ability to solve problems in unexpected ways, success in identifying patterns and relationships, enjoyment from posing original problems, a preference for working abstractly, rapid learning, a long concentration span when interested, a capacity for self-directed activity, a preference for mathematical activities, and enjoyment from mathematical puzzles and games (House, 1987). While many theorists propose similar characteristics to these (see Putt, 1998 for an overview), little is known about the cognitive differences that may distinguish a young spatially-gifted potential “Einstein” from a more analytically-gifted potential “Erdos”.

Catering for mathematically gifted students requires curriculum differentiation (D. Johnson, 1994; Sheffield, 1999). However, Hall (1997) argues that due to the inadequate curriculum guidance, teachers need to access and interpret research articles to modify the curriculum “to better fit the precociousness of the mathematically talented” (p. 21). The Senate Inquiry (Collins, 2001) also acknowledges the inadequacy of curriculum materials for the gifted. Given the need for teachers to access research information, to differentiate the curriculum for these students, the adequacy and accessibility of the literature base on educating the mathematical gifted is paramount.

Mathematically gifted students are affected in multiple ways by the attitudes of others. These students’ interests and capabilities set them apart from other children and identify them as a “marked” group or “deviant” population, due to the generally negative attitudes towards mathematics that are held by the general populace (Damarin, 2000). As
society finds an orientation towards mathematics less acceptable for girls than for boys, gifted girls are considered to be “doubly marked” (Damarin, 2000). Negative community attitudes towards the gifted are reflected by the derisive labelling of these students as “little Einsteins” or “nerds”. The attitudes of teachers towards gifted students vary substantially. At one extreme, there are teachers who are highly supportive of gifted students or who believe that all students should receive support to achieve their potential (McLeod & Copley, 1989). At the other extreme, there are teachers who view gifted students as over endowed and react negatively to any special treatment for these students or teachers who are either indifferent or unaware of the needs of these children. The most significant factor affecting teachers’ attitudes to gifted students is whether or not they have done any specific study in gifted education (Plunkett, 2000). Gifted students in Australia are further affected by the generally anti-intellectual perspective of the community and an overly broad view of giftedness that would be untenable internationally (Gross, 1993). Thus, although mathematically gifted students have characteristics that predispose them towards high performance and creative achievement, they need considerable support and resilience to overcome negative attitudes and fulfil their potential.

Thus, emerging from the literature about the education of mathematically gifted students are two fundamental questions that need to be addressed to inform teacher education, to underpin classroom practice in Australasia, and to guide future research:

1. What is the status of research on mathematically gifted students?
2. What are the characteristics of mathematically gifted students and how can their educational needs be supported?

Design and Methods

The two research questions about the education of the mathematically gifted students were addressed using a two-phase design. In phase 1, we undertook a document analysis (Hodder, 2000) to investigate the question, What is the status of research on mathematically gifted students? The documents selected for review were the past decade of Mathematics Research Group of Australasia Conference [MERGA] Proceedings and the past decade of issues of the Australasian Journal of Gifted Education [AJGE]. These publications were deemed to be representative of Australasian mathematics education research and gifted education research respectively and relatively accessible to teacher educators and classroom teachers. The abstracts or introductions of all full papers in the MERGA proceedings between 1992 and 2001 inclusive were read and papers selected for analysis in which reference was made to research with mathematically able students. Key words used in the search included “gifted”, “talented”, “high achievers”, “capable”, and “able”. Similarly, the papers in AJGE were reviewed and papers selected in which there was reference to “mathematics” or “numeracy” in the abstract or introduction. In phase 2, we employed a retrospective analysis of selected studies from our research (Torrance, 1994) to explore the question, What are the characteristics of mathematically gifted students and how can their educational needs be supported? Over the past decade, our research has encompassed a variety of studies on mathematically and scientifically gifted students. The research has generally involved teaching interventions of approximately 10 weeks duration that explored the characteristics, behaviours, and needs of gifted students. Typically, the data comprised classroom video recordings, field notes, students’ work samples, and interview responses. Although these studies varied in specific purpose, the studies referenced in this paper were underpinned by the latter research question.
Results and Discussion

The Status of Mathematically Gifted Students

The document analysis revealed a paucity of Australasian research on mathematically gifted students as reported in either MERGA proceedings or AJGE. Between 1992 and 2001, of the 794 published papers in the MERGA proceedings only four papers focussed on mathematically gifted students (Clarke & Bana, 2001; Curran, Daniel, & Holton, 1995; Marshall, 1994; Putt, 1998). In contrast, 22 publications focused on low achieving students or students with learning difficulties. An additional four papers addressed high and low achieving students or streaming. In AJGE, six papers of the 105 papers published in the past decade considered mathematically gifted students (Beardon, Jared, & Way, 1999; Hall, 1997; Landvoigt, Leder, & Forgasz, 1998; Lowrie, 1995, 1996; Taylor, 1992). Given that mathematically gifted students are the focus of less than one percent of publications in the MERGA proceedings and six percent of articles in AJGE, the research or dissemination of research on mathematically gifted students, has been of low status in the past decade. Although Australasian researchers in this area publish elsewhere (e.g. Diezmann & Watters, 2001), this literature may be inaccessible to many teachers.

Supporting the Mathematically Gifted Students

We explored the second question: What are the characteristics of mathematically gifted students and how can their educational needs be supported? through four sub-questions:

(a) What are the differing characteristics of these students? The central characteristic of mathematically gifted students is their advanced capacity to reason either analytically or spatially (Diezmann, 2001; Diezmann & Watters, 1996, 2000c). Analytically gifted students are generally fast and accurate workers, who are able to articulate their chain of reasoning. In contrast, spatially gifted students may underachieve in classrooms due to the typical emphasis on analytical tasks and may experience significant difficulty verbalising their reasoning (Diezmann & Watters, 1996). Spatially gifted students perform best on visual-spatial mathematical tasks and when using spatial tools and techniques, such as diagrams and visualisation. Hence, the identification of these students is dependent on providing relevant opportunities for them to demonstrate their ability. Spatially gifted students also tend to have interests in spatially-oriented activities, such as drawing, map reading, construction or chess. However, despite the tendency for spatially gifted students to underachieve in school, it is acknowledged that high spatial ability rather than high analytical ability generally underpins creative breakthroughs in mathematics and science (Diezmann & Watters, 2000c). The spatially gifted seem to be advantaged by being able to process information simultaneously rather than sequentially (Watters & English, 1995). Hence, they are able to abstract rich networks of connections between concepts and procedures, and capitalise on cognitive tools, such as analogies. Students who process information simultaneously are more successful on reasoning tasks than students who process information sequentially (Watters & English, 1995). Just as there is a need for sensitivity to the types of tasks that will allow spatially-gifted students to demonstrate their ability, there is also a need for appropriate tasks and procedures to identify other known underachieving populations, such as indigenous students (Cronin & Diezmann, submitted).

(b) What types of academic tasks have learning potential for these students? Mathematically gifted students’ capabilities and interests in mathematics can be many
years ahead of their age peers. These students seem to benefit most from four types of tasks. First, gifted students require challenging tasks to provide scope for learning and the use of metacognitive skills. Tasks that are too simple for particular students can be modified to increase the level of challenge. A task can be problematised by including more complex numbers in the task, by adding obstacles to solution, by requiring students to engage in novel solution processes, or by requiring students to use particular representations (Diezmann & Watters, 2000b). In contrast to many of their age peers, gifted students express an explicit preference for difficult mathematical tasks (Diezmann & Watters, 2000b). Second, gifted students may need tasks that introduce them to mathematical ideas beyond those typically addressed for their age group. For example, while most children in their first year at school learn about one digit numbers, similarly-aged gifted students may seek to develop multi-digit number sense to understand the quantitative information they encounter about topics such as space travel (Diezmann & English, 2001). Third, open-ended investigatory tasks can provide rich learning opportunities for gifted students. These tasks require the application of mathematical knowledge, can be cross-disciplinary, and provide scope for creativity (Diezmann, English, & Watters, 2001). Fourth, the tasks should be of interest to mathematically gifted students whose interests can differ substantively from their age peers. Young mathematically gifted children’s interests can include large numbers, space travel, futures, abstract mathematical explorations, and applications of mathematics (Diezmann & English, 2001; Watters & Diezmann, 1997). Fundamentally, a gifted student’s capability should guide the selection and implementation of tasks rather than a predominantly age-focused curriculum.

(c) What constitutes an effective learning environment for these students? There are three main features of an effective learning environment for mathematically gifted students apart from academic tasks and the role of the teacher, which have been addressed separately. First, the environment should provide opportunities for these students to develop the skills to become autonomous learners (Diezmann & Watters, 2000a). This includes knowing how to conduct investigations (Diezmann, English, & Watters, 2001) and how to learn through discourse (Diezmann & Watters, 2001). Productive discourse incorporates evidence, logic, and argumentation and involves students in sharing ideas, building on each other’s ideas, and critiquing ideas (Diezmann & Watters, 1998, 2000a, 2001). Second, gifted students’ preferences for working individually or in a group should be addressed. These students’ preferences for working in a group situation appear to be goal-oriented (Diezmann & Watters, 2001). When gifted students work on relatively easy tasks, they prefer to work independently or side-by-side with another student. However, when tasks are sufficiently challenging, gifted students’ prefer to work in a group in order to share knowledge and access a support network (Diezmann & Watters, 2001). Third, gifted students should also have opportunities to work with like-minded peers who share their interests and will challenge their ideas (Diezmann & Watters, 2001). This may occur within the regular classroom or through acceleration or enrichment. Acceleration may benefit students who are sufficiently advanced to work full-time at a level beyond that of their age peers (Diezmann, Watters, & Fox, 2001), while enrichment programs enable students to explore topics in depth with like-minded peers (Watters & Diezmann, 2000).

(d) How can teachers support these students? Teachers can support mathematically gifted students in three key ways. First, teachers need to educate themselves sufficiently about gifted students (Diezmann & Watters, 1999). This includes ability to identify mathematically gifted students including spatially gifted students (Diezmann & Watters, 2000c), and indigenous students (Cronin & Diezmann, submitted); being aware of the
difficulties of gifted students (Diezmann & Watters, 1996); and knowing effective strategies for catering for gifted students (Watters & Diezmann, 1997). Teachers also need to be aware of mandated policies on the educational provisions for gifted students and their subsequent responsibilities for the education of these students (Diezmann, Watters, & Fox, 2001). As many teachers are untrained in gifted education, strategic professional development support is necessary (Holz, Diezmann, & Watters, 1999). Second, teachers should select challenging tasks for these students and provide the necessary support for students to learn from these tasks. This support includes modifying the difficulty level of simple tasks (Diezmann & Watters, 2000b), providing authentic opportunities for students to benefit from working collaboratively with other gifted students (Diezmann & Watters, 2001), providing the optimal scaffolding to assist students on challenging tasks (Diezmann & Watters, 2000b), enabling them to learn from discourse (Diezmann & Watters, 2001), and providing hands-on materials or pictorial aids where necessary to stimulate or facilitate thinking (Diezmann & English, 2001). Third, teachers should support the development of students’ mathematical knowledge and reasoning ability (Diezmann, 2001) because these are the cornerstones of creative achievement in mathematics (Diezmann, 2001). Fourth, teachers should consider how new syllabi impact on gifted students. For example, in science, while the philosophy of outcome-based education has scope for gifted students, but enactment may disadvantage students (Diezmann & Watters, 2002).

Conclusion

Clearly, it is important to educate the mathematically gifted students so they can be fulfilled as individuals and contribute to society. At present, there are considerable funds being expended on the education of these students through special initiatives (e.g., Education Queensland, 2002). However, there appears to be limited research-based literature on educating mathematically gifted students in readily accessible mathematics education research literature or the counterpart gifted literature to inform these initiatives, or to guide classroom teachers and teacher educators. While our work provides some insight into the education of mathematically gifted students, this review has raised our awareness of the need for more strategic research that brings together theoretical frameworks from both mathematics and gifted education. Additionally, the literature base on mathematically gifted students can be developed by researchers reporting the performance of gifted students as a distinct cohort within their studies. Furthermore, there is a need for a concerted research effort on the education of mathematically gifted students particularly, in relation to commonly held views and practices. To adequately cater for mathematically gifted students, we need to draw upon and extend research that informs professionals, curriculum developers and policy makers about educational practices and addresses the assumptions underpinning the education of the mathematically gifted. Our research with these students indicates that many common practices and views about the education of the mathematically gifted are merely myths, and hence, have little educational validity. The myths are:

1. **Mathematically gifted students are high achievers and, fast and accurate workers.**
   While this is true of some mathematically gifted students, there are also gifted students who underachieve on traditional school mathematics tasks.

2. **Additional exercises or activities for “fast finishers” caters for the needs of mathematical gifted students.** If these tasks are only at the same level as the class work, this work is merely “busy work” and lacks the opportunity for mathematical learning.
3. **Mathematics competitions cater for the needs of mathematically gifted students.** While students can demonstrate their ability and compete at a high level periodically in competitions, they need ongoing opportunities to develop their potential.

4. **Gifted students prefer to work in homogeneous groups.** Students’ preferences for working individually or in a group vary according to the challenge of the task and their need for cognitive and affective support.

5. **Gifted students do not need opportunities to work with other gifted students.** Gifted students interests’ and abilities can vary substantively from their age peers, hence without the company of other gifted students or older students, they can be very isolated and have limited opportunities to learn through discourse.

6. **Gifted students need little teacher support.** Gifted students need little support on routine tasks. However, to maximise learning they need support on challenging tasks.

7. **There are insufficient funds and time to dedicate to the education of mathematically gifted students.** This view is short-sighted and overlooks the long-term return that a small investment in the education of mathematically gifted students should yield. The current shortage of mathematics teachers may be a consequence of past failures to nurture and support gifted students’ interests in mathematics.

### References


